Technical Comments

Comment on "Relative Variation of Rocket Vehicle Specific Impulse and Structural Weight Fraction for Constant Burnout Velocity"

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Nomenclature

e = Napierian logarithmic base, 2 718
g_0 = mass conversion factor
Isp = specific impulse
s = ratio of structural weight to initial gross weight
l = ratio of payload weight to initial gross weight
V_{bo} = vehicle flight velocity at end of burning
Ve = exhaust velocity
Vo = flight velocity
η_p = propulsive efficiency

IN Ref 1, which is concerned with optimum rocket flight performance, a discussion of the relative influence of changes in propulsion system design or propellant capability is presented An equation, (3 23b) of Ref 1, which expresses the ratio $\Delta Isp/\Delta s$, is derived for conditions of constant burnout velocity and zero-g flight in vacuum The final equation is in error, as are the curves of Fig 3-9, p 3-15 of Ref 1 The correct results are derived by writing the equation for burnout velocity

$$V_{bo} = g_0 Isp \ln[1/(s + l)] \tag{1}$$

Then

$$\frac{dV_{bo}}{ds} = -g_0 Isp \left(\frac{1}{s + l}\right) + g_0 \ln\left(\frac{1}{s + l}\right) \frac{d\,Isp}{ds} \tag{2}$$

For constant V_{bo}, $dV_{bo}/ds = 0$ Then Eq (2) gives

$$\frac{d\,Isp}{ds} = Isp \left(\frac{1}{s + l}\right)\left\{\frac{1}{\ln[1/(s + l)]}\right\} \tag{3}$$

or

$$\frac{\Delta Isp}{\Delta s} = -Isp \left[\frac{1}{(s + l)\; ln(s + l)}\right] \tag{4}$$

Equation (4) is the corrected version of Eq (3-23b) of Ref 1 The variation of $\Delta Isp/\Delta s$ with $s + l$ is presented in Fig 1 in the manner of Fig 3 9 of Ref 1 As the rocket vehicle becomes more structurally efficient, i e , small values of $s + l$, the conclusion of Ref 1 is correct: large increases in specific impulse can be offset by small fractional increases in structural weight However, as the vehicle becomes structurally inefficient (high values of $s + l$) the curves of constant specific impulse exhibit a positive increasing slope rather than being asymptotic to the $s + l$ axis[1] as $s + l$ approaches 1 0 Thus, the same conclusion is true for high values of the structural weight fraction

The minimum points on the curves obviously occur at the value of structural fraction which will allow the maximum variation in structural weight fraction for the least required change in specific impulse Equation (3) may be differentiated to give

$$\frac{d^2Isp}{ds^2} = -Isp \left[\frac{-[\ln(s + l) + 1]}{[(s + l)\ln(s + l)]^2}\right] \tag{5}$$

At the minimum point $d^2 Isp/ds^2 = 0$ and, thus

$$s + l = 1/e = 0\ 368 \tag{6}$$

for minimum $\Delta Isp/\Delta s$ Substitution of $s + l = 1/e$ in Eq (1) gives

$$V_{bo} = g_0\, Isp \ln e = g_0\, Isp \tag{7}$$

The burnout velocity is then equal to the theoretical nozzle exit velocity ($V_{bo} = Ve = g_0\, Isp$)

It is interesting to note that the rocket propulsive efficiency $\eta_p{}^2$ is a maximum at this condition since

$$\eta_p = \frac{2Vo/Ve}{1 + (Vo/Ve)^2} \tag{8}$$

and $\eta_p \rightarrow 1\ 00$ for $V_{bo} = Ve = Vo$ It is concluded that the maximum permissible variation in structural weight fraction with the minimum required change in propellant capability (Isp) occurs at values of vehicle mass ratio in the neighborhood of $e = 2\ 718$, which is coincident with maximum pro pulsive efficiency

References

[1] Summerfield, M and Seifert, H S , "Flight performance of a rocket in straight line motion," *Space Technology*, edited

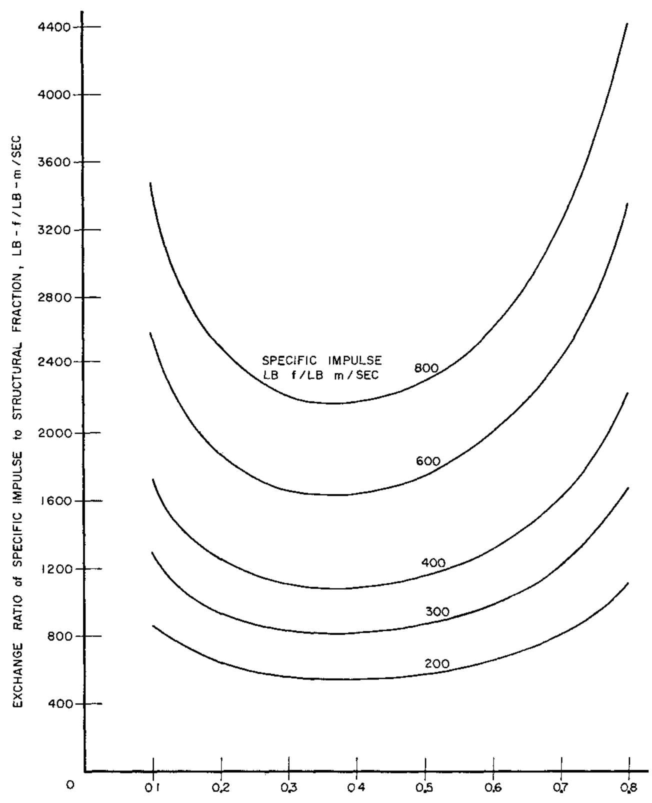

Fig 1 Exchange ratio curves for $\Delta Isp/\Delta s$, zero-g vacuum flight

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by H S Seifert (John Wiley and Sons, Inc , New York, 1959), Chap 3, pp 3-14–3-15

[2] Sutton, G P , *Rocket Propulsion Elements* (John Wiley and Sons, Inc , New York, 1956), 2nd ed , p 19

Slip Flow and Hypersonic Boundary Layers

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The problem of the hypersonic similar laminar boundary layeı is modified to include first order slip boundary conditions It is observed that the inclusion of these new boundary conditions result in no change in the energy transfer to the wall from the no-slip values, implying that slip effects alone aıe not adequate to explain the appearance of the heat tıansfer "plateaus" observed near the leading edge of a sharp flat plate in hypersonic flow In addition, it is shown how the inclusion of slip boundary condition as the only higher order effect results in complete disagreement with observed pressure measurements for the cold-wall case

In a recent note, Talbot[1] suggests a criterion for slip flow near the sharp leading edge of a flat plate in hypersonic flow It is significant that the parameter suggested by Talbot as a suitable correlating parameter for heat transfer and induced pressure experiments can be interpreted both as the ratio of the mean free path behind the shock wave to the boundary layer thickness, as well as the ratio of the leading edge shock thickness to the distance from the leading edge Although Talbot's correlating parameter is not really the appropriate one which arises from the inclusion of slip boundary conditions to the traditional strong hypersonic viscous interaction problem, it can, of course, be related to that parameter The author has included first-order slip boundary conditions in the calculation of heat transfer for that case and has arrived at the not unexpected conclusion that there is no effect of hypersonic laminar heat transfer of the inclusion of slip velocity and temperature jump boundary conditions This result, which was obtained earlier by Maslen[2] for a zero-pressure gradient no-slip flow is here extended to the class of hypersonic similar solutions characterized by u = const, $p \sim x^n$ These solutions, which were tabulated by Dewey,[3] include the conventional no-slip strong interaction problem where $p \sim x^{-1/2}$ Our negative result is restricted only to the heat transfer to the wall Induced pressure and skin friction, on the other hand, in general will exhibit a first-order slip effect Thus the possibility is raised that additional higher order effects besides slip will be necessary to account for the observed pressure and heat transfer "plateaus "

At the present time, it is not clear just how a systematic approach to the solution of the Navier-Stokes equation should proceed for this geometry, and there is even the possibility that coupling between the viscous boundary layer and the shock structure may not be very well represented by the Navier-Stokes equation It is felt that it is inappropriate to attribute to slip flow and rarefaction effects which may well be within the scope of the Navier-Stokes equations

It is quite apparent from our results that slip flow alone is incapable of reducing the energy transfer to the wall from the usual strong interaction result Perhaps the success of Talbot's parameter in correlating both induced pressure and heat transfer measurements hinges on its dual significance as an estimator of the effects of shock-structure viscous flow field coupling, as well as an estimator of slip effects

Analysis

Consider a perfect gas, constant Prandtl number, and arbitrary viscosity-temperature dependence The flow external to the boundary layer is presumed to be hypersonic, corresponding to the limit $U\delta^2/2H\delta \to 1$ The notation of Hayes and Probstein[4] is used throughout

The form of the first-order slip boundary conditions in the η, ξ variables suggests that the appropriate expansion parameter for first-order slip effects is $\epsilon \equiv \lambda_w \rho_w U\delta/(2\xi_0)^{1/2}$ For the hypersonic no-slip similar flows corresponding to $U\delta$ = const, $p_0 \sim x^n$, this parameter ϵ varies as $x^{-(n+1/2)}$ Consider a first-order expansion in ϵ of f, g, p ξ, and $\rho\mu/\rho_w\mu_w$:

$$\left.\begin{aligned} f &= f_0(\eta) + \epsilon f_1(\eta) \\ g &= g_0(\eta) + \epsilon g_1(\eta) \\ p &= p_0[1 + \epsilon p_1] \\ \xi &= \xi_0[1 + 2\epsilon p_1] \\ \rho\mu/\rho_w\mu_w &\equiv (\mu/T)\,(T_w/\mu_w) \equiv N \\ N &= N_0 + \epsilon \left.\frac{T_w}{g_w}\frac{dN}{dT}\right)_{T=T_0} (g_1 - 2f_1'f_0') \end{aligned}\right\} \quad (1)$$

If the effect of the self-induced pressure gradient is to be calculated, then p_1 will be determined by a suitable relationship between the displacement thickness and the pressure Otherwise it may be set identically equal to zero In the analysis of the heat transfer, which will be given below, it is quite apparent that its magnitude does not enter into consideration at all It does, however, enter into the momentum equation as a coefficient of an inhomogeneous term and consequently will be involved in the skin friction and in the solution for f_1 and g_1

The first-order boundary layer equations with appropriate boundary conditions become

$$\frac{\partial}{\partial\eta}\left\{N_0 f_1'' + \frac{T_w}{g_w}\left(\frac{dN}{dT}\right)_{T=T_0}(g_1 - 2f_0'f_1')\,f_0''\right\} + f_1 f_0'' + f_0 f_1'' + \beta_0[g_1 - 2f_0'f_1'] + \beta_1\beta_0(g_0 - f_0')^2 = \left[\beta_0\left(\frac{h\delta_0}{H\delta}\right) - 1\right][f_1'f_0' - f_1 f_0''] \quad (2a)$$

$$\frac{1}{Pr}\frac{\partial}{\partial\eta}\left\{N_0 g_1' + \frac{T_w}{g_w}\frac{dN_0}{dT}(g_1 - 2f_0'f_1')g_0'\right\} + f_0 g_1' + f_1 g_0' + 2\frac{\partial}{\partial\eta}\left\{N_0\left(1\ \frac{1}{Pr}\right)(f_1'f_0'' + f_0'f_1'')\right\} + 2\frac{\partial}{\partial\eta}\left\{\frac{T_w}{g_w}\left(\frac{dN_0}{dT}\right)_{T=T_0}(g_1 - 2f_0'f_1')\left(1 - \frac{1}{Pr}\right)(f_0'f_0'')\right\} = \left[\beta_0\left(\frac{h\delta_0}{H\delta}\right) - 1\right][g_1 f_0' - f_1 g_0'] \quad (2b)$$

The boundary conditions are $f_1'(0) = a_v f_0''(0)$, $g_1(0) = a_T g_0'$, where a is a velocity slip coefficient, a_T is a temperature jump coefficient, $f_1(0) = 0$, $f_1'(\infty) \to 1$, $g_1(\infty) \to 0$ Note that

$$\frac{2d\ln U_\infty}{d\ln\xi}\frac{H\delta}{h\delta} \equiv \beta_0(1 + \epsilon\beta_1) = \frac{\gamma - 1}{\gamma}\quad\frac{n}{n-1}\left(1 + \epsilon\frac{3}{2}p_1\right) \quad (3)$$

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